MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

20 YEARS
1998-2018

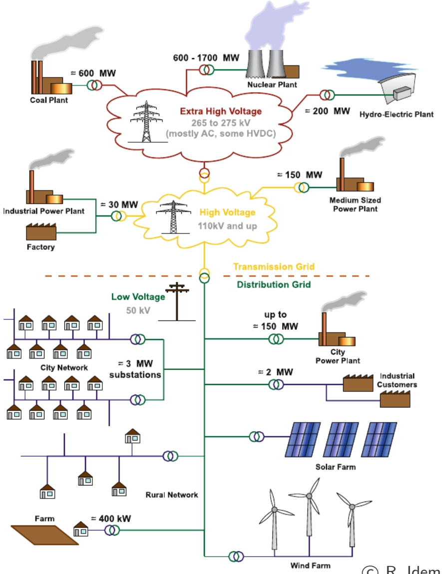
MS216: Mathematical Methods for Control and Optimization of Large-Scale Energy Networks

Manuel Baumann and Yue Qiu

February 27, 2019

SIAM Conference on Computational Science and Engineering
Spokane, Washington, USA

Aim of this mini-symposium



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The energy network is subject to ongoing changes:

- renewables
- electric car
- batteries
- power-to-gas

Gives rise to new mathematical challenges!

Today's schedule

Part I/II

Manuel Baumann	Model-reduction for Dynamic Power Flow
Domenico Lahaye	Newton-Krylov Methods for the PFE
Riccardo Morandin	Hierarchical Modeling of Power Networks
Baljinnyam Sereeter	Four Mathematical Formulations of the OPF

Today's schedule

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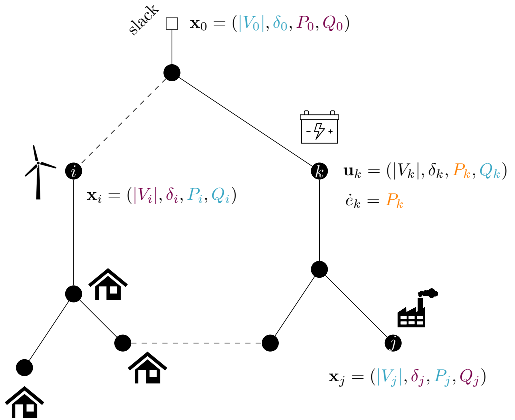
Part II/II at Room 302A (2:15 PM – 3:55 PM)

Peter Benner ~~Yue Qiu~~	Numerical Methods for Gas Networks
Stephan Gerster	Fluctuations in Supply Networks
Jennifer Uebbing	Optimization of Power-to-methane Processes
Anne Markensteijn	Load Flow Analysis of Multi-carrier Energy Systems

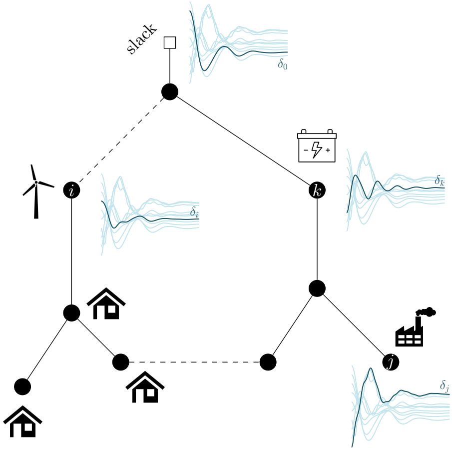
Model-order Reduction for Dynamic Power Flow Simulations



Power flow equations

Nonlinear relation between the voltage $V_i = |V_i|e^{-j\delta_i}$ and the power $S_i = P_i + jQ_i$ at node i.

Dynamic power flow simulation



Power flow equations

Nonlinear relation between the voltage $V_i = |V_i|e^{-j\delta_i}$ and the power $S_i = P_i + jQ_i$ at node i.

Swing equations

ODE's on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$,

$$\ddot{\delta}_i + \dot{\delta}_i = P_i - \sum_{j \neq i} \sin(\delta_i - \delta_j)$$

[coefficients omitted.]

Dynamic power flow simulation

Three leading models

Governing equations (*swing equations*) at network node $i \in \mathcal{V}$,

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = A_i - \sum_{j \neq i} K_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}), \quad i = 1, ..., N,$$

yield for a specific choice of parameters the three models

EN **effective network model** ($N = |\mathcal{V}_{\text{gen}}|$),

SM **synchronous motor model** ($N = |\mathcal{V}|$),

SP structure-preserving model ($N > |\mathcal{V}|$).

T. Nishikawa and A. E. Motter (2015). *Comparative analysis of existing models for power-grid synchronization.* New Journal of Physics 17:1.

Linear MOR in a nutshell

A linear dynamical system,

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x}$$

is approximate by a reduced-order model,

$$\dot{\hat{\mathbf{x}}} = \hat{A}\hat{\mathbf{x}} + \hat{B}\mathbf{u}$$
$$\hat{\mathbf{y}} = \hat{C}\hat{\mathbf{x}}$$

such that the output difference $\|\mathbf{y} - \hat{\mathbf{y}}\|$ is small.

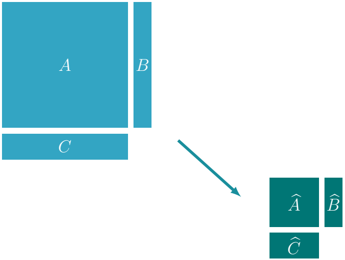


Figure: Petrov-Galerkin projections $\hat{A} := W_r^T A V_r$, $\hat{B} := W_r^T B$, and $\hat{C} := C V_r$.

Swing equations are **non**linear,

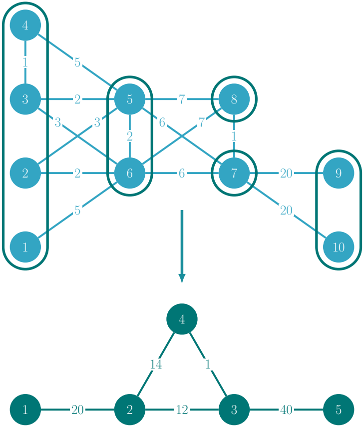
$$\dot{x} = f(x,t), \quad x := [\delta, \dot{\delta}].$$

Projection/reduction:

$$\dot{\hat{x}} = W_r^T f(V_r \hat{x}, t).$$

Nonlinear MOR:

- $f(V_r \hat{x})$ still large
- hyper-reduction
- clustering: $V_r = W_r = P(\pi)$



Algorithm:

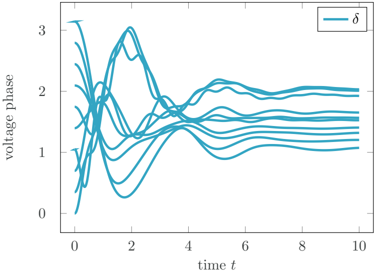
1. Collect snapshots

$$X := [\mathbf{x}(t_1), ..., \mathbf{x}(t_s)]$$

2. Principal components

$$X =: U\Sigma V^T \quad \leftarrow \text{SVD of } X$$

3. k-means clustering
4. Projection $P(\pi)$



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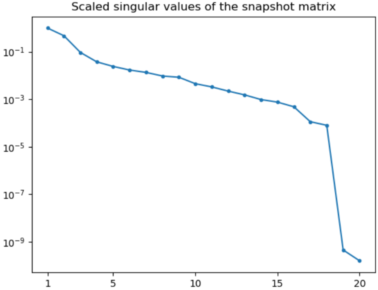

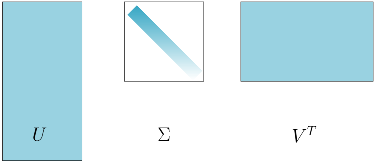

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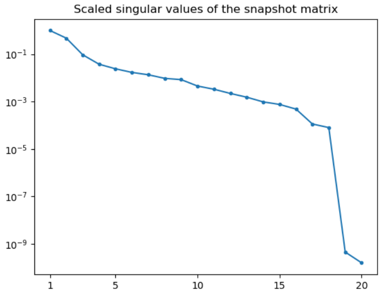

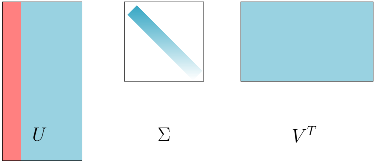

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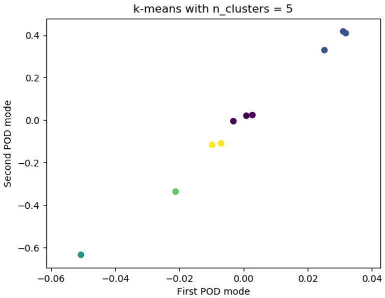

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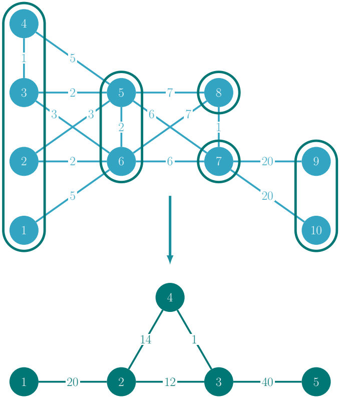
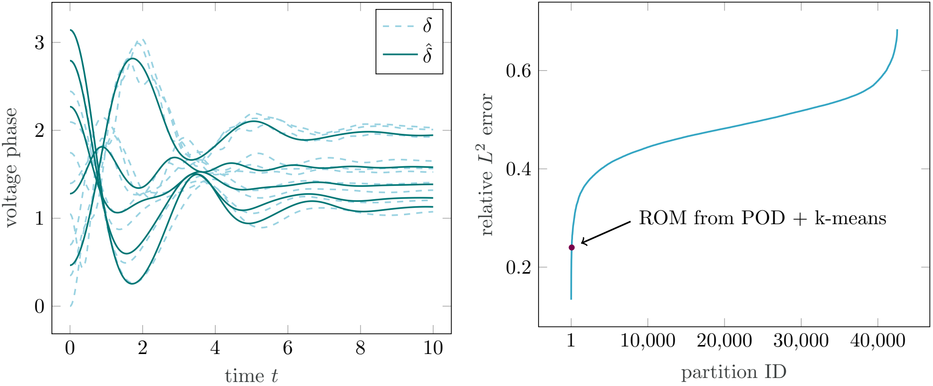


The approximation error:



The (simplified) swing equations,

$$H_i\ddot{\delta}_i + D_i\dot{\delta}_i = A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j), \quad i = 1, ..., N,$$

in vectorized form, yields a structure-preserving ROM:

$$H\ddot{\boldsymbol{\delta}} + D\dot{\boldsymbol{\delta}} = A - \left(K \odot \sin(\boldsymbol{\delta}\mathbf{1}_n{}^T - \mathbf{1}_n\boldsymbol{\delta}^T)\right)\mathbf{1}_n, \qquad \text{FOM}$$

$$\hat{H}\ddot{\hat{\boldsymbol{\delta}}} + \hat{D}\dot{\hat{\boldsymbol{\delta}}} = \hat{A} - \left(\hat{K} \odot \sin(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)\right)\mathbf{1}_r. \qquad \text{ROM}$$

Model-Order Reduction on Graphs

The (simplified) swing equations,

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$$\hat{H}\ddot{\hat{\boldsymbol{\delta}}} + \hat{D}\dot{\hat{\boldsymbol{\delta}}} = \hat{A} - \left(\hat{K} \odot \sin(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)\right)\mathbf{1}_r. \qquad \text{ROM}$$

The nonlinear term reduces to:

$$\begin{aligned} P^T\left(K \odot \sin(\boldsymbol{\delta}\mathbf{1}_n{}^T - \mathbf{1}_n\boldsymbol{\delta}^T)\right)\mathbf{1}_n &\approx P^T\left(K \odot \sin(P\hat{\boldsymbol{\delta}}(P\mathbf{1}_r)^T - P\mathbf{1}_r(P\hat{\boldsymbol{\delta}})^T)\right)P\mathbf{1}_r \\ &= P^T\left(K \odot \sin(P(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)P^T)\right)P\mathbf{1}_r \\ &= \left(P^TKP \odot \sin(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)\right)\mathbf{1}_r \end{aligned}$$

Balanced truncation for a quadratic re-formulation of the swing equations

Reformulation to quadratic system

Swing equations in first-order form, $\omega := \dot{\delta}$,

$$\begin{bmatrix} \dot{\delta}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} \omega_i \\ \frac{\omega_R}{2H_i} \left(A_i - \sum_{j \neq i} K_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}) - \frac{D_i}{\omega_R} \omega_i \right) \end{bmatrix},$$

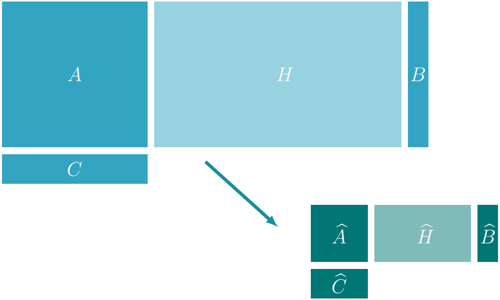
Quadratic formulation introducing $s_i := \sin(\delta_i)$ and $c_i := \cos(\delta_i)$,

$$\begin{bmatrix} \dot{\delta}_i \\ \dot{\omega}_i \\ \dot{s}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} \omega_i \\ \frac{\omega_R}{2H_i} \left(A_i - \sum_{j \neq i} K_{ij} (s_i c_j \gamma_{ij}^c - c_i s_j \gamma_{ij}^c - c_i c_j \gamma_{ij}^s - s_i s_j \gamma_{ij}^s) - \frac{D_i}{\omega_R} \omega_i \right) \\ c_i \omega_i \\ -s_i \omega_i \end{bmatrix},$$

with constants $\gamma_{ij}^c := \cos(\gamma_{ij})$ and $\gamma_{ij}^s := \sin(\gamma_{ij})$.

Input/output systems in quadratic form

$$\dot{x}(t) = Ax(t) + H\; x(t) \otimes x(t) + Bu(t),$$
$$y(t) = Cx(t), \quad x(0) = x_0.$$



Balanced truncation for quadratic systems

Consider a quadratic input/output system,

$$\dot{x}(t) = Ax(t) + H\ x(t) \otimes x(t) + Bu(t),$$
$$y(t) = Cx(t), \quad x(0) = x_0.$$

with $x_i := [\delta_i, \xi_i, s_i, c_i]$ and represented by matrices $\{A, B, C, H\}$.

Projection-based model-order reduction

Requires solution of two quadratic matrix equations

- $A\mathbf{P} + \mathbf{P}A^T + H(\mathbf{P} \otimes \mathbf{P})H^T = -BB^T, \qquad \mathbf{P} =: RR^T,$
- $A^T\mathbf{Q} + \mathbf{Q}A + H^{(2)}(\mathbf{P} \otimes \mathbf{Q})(H^{(2)})^T = -C^TC, \quad \mathbf{Q} =: SS^T.$

Obtain projection spaces based on truncated SVD of S^TR.

P. Benner, P. Goyal (2017). *Balanced Truncation Model Order Reduction For Quadratic-Bilinear Control Systems.* arXiv:1705.00160.

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Projection-based model-order reduction

Requires solution of two quadratic matrix equations

- $A_s \mathbf{P} + \mathbf{P} A_s{}^T + H(\mathbf{P} \otimes \mathbf{P})H^T = -BB^T, \qquad \mathbf{P} \approx RR^T,$
- $A_s{}^T \mathbf{Q} + \mathbf{Q} A_s + H^{(2)}(\mathbf{P} \otimes \mathbf{Q})(H^{(2)})^T = -C^T C, \quad \mathbf{Q} \approx SS^T.$

Obtain projection spaces based on truncated SVD of $S^T R$.

P. Benner, P. Goyal (2017). *Balanced Truncation Model Order Reduction For Quadratic-Bilinear Control Systems.* arXiv:1705.00160.

Numerical experiments

Aim: Consider slack node dynamics as output, $y = Cx = x_1$, and isolate node x_1 when clustering.

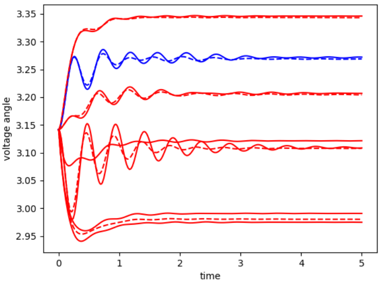


Figure: EN model clustering-based MOR.

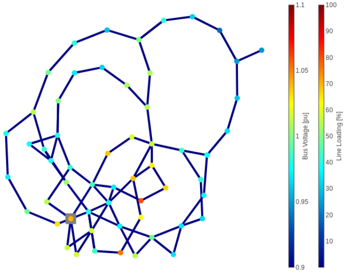
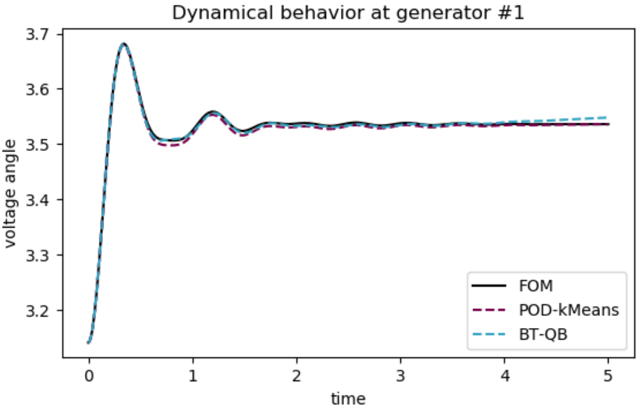


Figure: `IEEE Case57` taken from MATPOWER.

Numerical experiments



case57.m	FOM dim.	ROM dim.	rel. L_∞ error
EN model	14 vs. 28	10 vs. 19	0.0030 vs. 0.0016
SM model	114 vs. 228	36 vs. 175	0.0024 vs. 0.0032

Summary

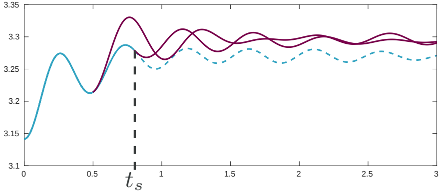
Conclusions:

- Structure-preserving ✓
- Stability-preserving ✗
- state-lifting in QB case

Future work:

- Third-order swing equation, i.e. $|V_i| \rightsquigarrow |V_i(t)|$
- Fault recovery: line failure at time $t = t_s$ yields a switched system in quadratic form, i.e.

$$\{A_1, B_1, C_1, H_1\} \overset{t_s}{\rightsquigarrow} \{A_2, B_2, C_2, H_2\}.$$



Selected references

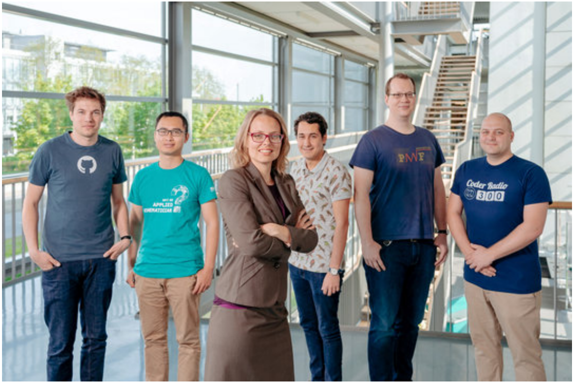
P. Benner and P. Goyal.
Balanced truncation model order reduction for quadratic-bilinear systems.
e-prints 1705.00160, arXiv, 2017.

P. Mlinarić, T. Ishizaki, A. Chakrabortty, S. Grundel, P. Benner, and J.-i. Imura.
Synchronization and aggregation of nonlinear power systems with consideration of bus network structures.
In *Proc. European Control Conf. (ECC)*, 2018.

F. Weiß.
Simulation, analysis, and model-order reduction for dynamic power network models.
Master's thesis, Otto-von-Guericke University Magdeburg, 2019 (ongoing).

The SES team at MPI Magdeburg

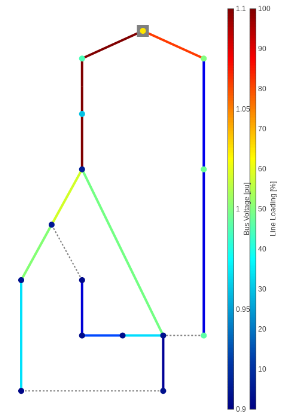
Thank you for your attention.
Question?



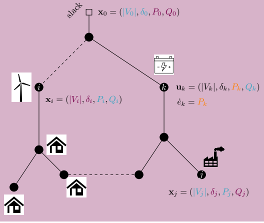
http://konsens.github.io

Optimization of batteries

Test network



Mathematical model



At the nodes:

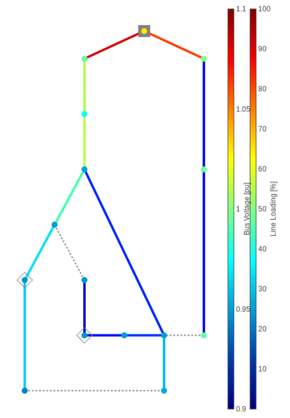
- voltage $V_i = |V_i|e^{j\delta_i}$
- power $S_i = P_i + jQ_i$
- batteries u_k

Optimization problem

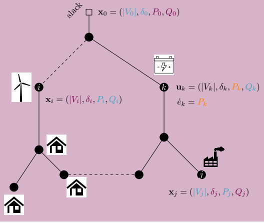
$\min\limits_{u}$	$\int_{t_0}^{t_e} \ell(V(u), t)\,\mathrm{d}t$	(line loading)
s.t.	$c_E(V, S, t) + Bu(t) = 0$	(equal. constr.)
	$\underline{x} \leq V_i(t), S_i(t) \leq \overline{x}$	(state const.)
	$\underline{u} \leq u_k(t) \leq \overline{u}$	(control const.)

Optimization of batteries

Test network



Mathematical model



At the nodes:

- voltage $V_i = |V_i| e^{j\delta_i}$
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- batteries u_k

Optimization problem

$$\min_u \int_{t_0}^{t_e} \ell(V(u), t)\, \mathrm{d}t \qquad \text{(line loading)}$$

$$\text{s.t.} \quad c_E(V, S, t) + Bu(t) = 0 \qquad \text{(equal. constr.)}$$

$$\underline{x} \le V_i(t), S_i(t) \le \overline{x} \qquad \text{(state const.)}$$

$$\underline{u} \le u_k(t) \le \overline{u} \qquad \text{(control const.)}$$