International Conference On Preconditioning Techniques 2017

Sponsored by: Shell

Preconditioning the elastic wave equation in 2D and 3D based on inexact MSSS matrix computations

Manuel Baumann Yue Qiu Martin B. van Gijzen

July 31, 2017

TU Delft

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

Keywords: elastic wave propagation, Krylov subspace methods, preconditioning, multiple frequencies

Problem statement

The discretized **time-harmonic elastic wave equation** yields,

$$(K + i\omega_k C - \omega_k^2 M)x_k = b, \quad k = 1, ..., N, \tag{1}$$

where $\{\omega_1, ..., \omega_N\}$ are a range of (angular) frequencies.

1. Linearization:

$$(\mathcal{K} - \omega_k \mathcal{M})\mathbf{x}_k := \left\{ \begin{bmatrix} iC & K \\ I & 0 \end{bmatrix} - \omega_k \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} \omega_k x_k \\ x_k \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

2. Apply shift-and-invert preconditioner:

$$\mathcal{P}(\tau)^{-1} := (\mathcal{K} - \tau\mathcal{M})^{-1} \qquad \text{[cf. Fig. 3 for choice of } \tau \in \mathbb{C}\text{]} \tag{2}$$

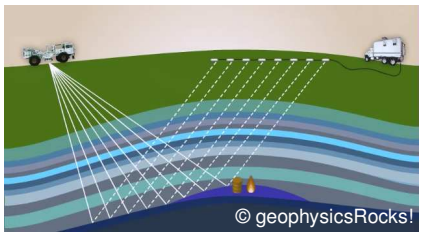

Fig. 1: Full-waveform inversion

Preconditioning the elastic operator in 2D and 3D

For the **2D operator** (red in Fig. 2) we compute a block-LU factorization of the form $\mathcal{P}(\tau) = LSU$, with

$$L_{i,j} = \begin{cases} I & \text{if } i = j \\ P_{i,j} S_j{}^{-1} & \text{if } i = j+1 \end{cases}, \quad U_{i,j} = \begin{cases} I & \text{if } j = i \\ S_i{}^{-1} P_{i,j} & \text{if } j = i+1 \end{cases},$$

and Schur complements given by the recursion,

$$S_i = \begin{cases} P_{i,i} & \text{if } i = 1 \\ P_{i,i} - P_{i,i-1} S_{i-1}{}^{-1} P_{i-1,i} & \text{if } 2 \le i \le n_x. \end{cases}$$

Here, the matrices S_i are SSS matrices; and inverses are computed *inexactly*, cf. [2].

For the **3D operator**, we consider a splitting $\mathcal{P}(\tau) = L + D + U$:

$$\mathcal{P}_h(\tau) = \underbrace{(LD^{-1} + I)D(D^{-1}U + I)}_{\text{"}n_z \text{ times a 2D problem"}} + \underbrace{P\mathcal{P}_H(\tau)^{-1}R}_{\text{"small 3D"}}$$

- The block SSOR preconditioner makes use of efficient 2D computations, cf. [1].
- Additive coarse grid correction yields grid-independent convergence ($H \gg h$).

MSSS matrix structure

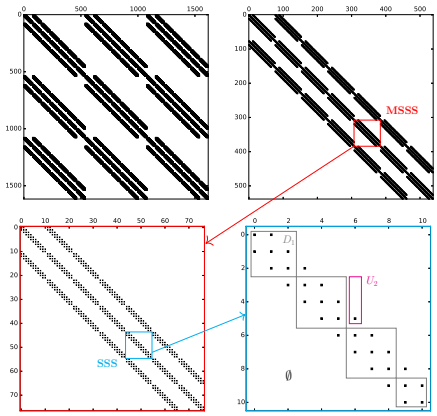

Fig. 2: MSSS structure of the 3D matrix (1)

Simulation results for 2D and 3D

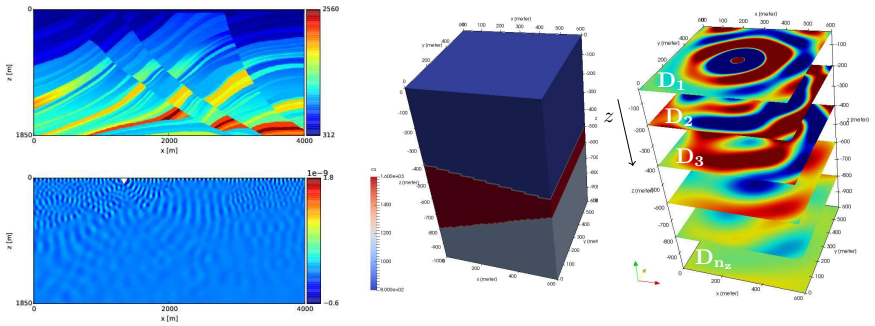

Fig. 4: Simulation results for the 2D `Marmousi-II` problem, and a 3D `wedge` problem

Preconditioned spectra

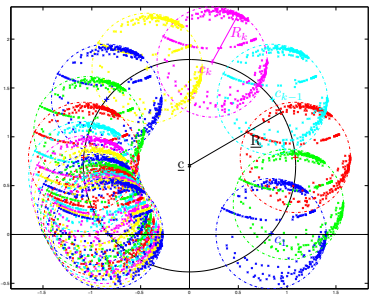

Fig. 3: Spectra after Möbius transformation. Choose τ such that $R_k/\|c_k\| \to \min$

References

[1] M. Baumann, R. Astudillo, Y. Qiu, E.Y.M. Ang, M.B. van Gijzen, and R.-É. Plessix (2017). *An MSSS-Preconditioned Matrix Equation Approach for the Time-Harmonic Elastic Wave Equation at Multiple Frequencies.* Springer Computat. Geosci., DOI: 10.1007/s10596-017-9667-7.

[2] Y. Qiu (2015). *Preconditioning Optimal Flow Control Problems Using Multilevel Sequentially Semiseparable Matrix Computations.* PhD Thesis, Delft University of Technology.