A Set of Fortran 90 and Python Routines for Solving Linear Equations with IDR(s)

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Induced Dimension Reduction (IDR) method

IDR(s) is a Krylov subspace method originally proposed to solve system of linear equations,

\[ Ax = b, \]

where the coefficient matrix \( A \) is large, sparse, and nonsymmetric. IDR(s) is a short-recurrence method which has obtained attention for its rapid convergence and computational efficiency.

IDR(s) for linear matrix equations

Generalization of the IDR theorem. Let \( A \) be a linear operator over a finite dimensional subspace \( D \) and \( I \) be the identity operator over the same subspace. Let \( S \) be any (proper) subspace of \( D \). Define \( G_0 \equiv D \). If \( S \) and \( G_0 \) do not share a nontrivial invariant subspace of the operator \( A \), then the sequence of subspaces \( G_j \), defined as

\[ G_j = (I - \omega_j A)(G_{j-1} \cap S), \quad j = 1, 2, \ldots, \omega_j \neq 0, \]

has the following properties for \( j \geq 0 \):

1. \( G_{j+1} \subset G_j \), and
2. \( \dim(G_{j+1}) < \dim(G_j) \) unless \( G_j = \{0\} \).

Using this generalization, we develop an IDR-algorithm for solving linear matrix equations of the form,

\[ \sum_{j=1}^{k} A_j X B_j^T = C. \]

IDR(s) for shifted linear systems

For the efficient solution of shifted linear systems,

\[ (A - \sigma_k I)x_k = b, \quad k = 1, 2, \ldots, \]

two IDR variants have recently been developed:

- Multi-shift QMRIDR(s) relies on a generalized Hessenberg decomposition, cf. [4],
- MSIDR(s) generates collinear residuals such that \( r_j^{(\sigma_k)} \in G_j, \forall k \), cf. [3].

The latter can be used as a preconditioner in a nested algorithm.

Software features

IDR implementation developed at TU Delft:

- Standalone implementation in Fortran 90 and Python
- Flexible user interface via types
- Advanced features, e.g. subspace recycling, Ritz values
- Solving matrix equations and multiple right-hand sides:

\[
\begin{align*}
\text{>> } & \text{ A } = \text{numpy.random.rand(n,n)} \\
\text{>> } & \text{ S } = \text{numpy.diag(sigma)} \\
\text{>> } & \text{ B } = \text{numpy.random.rand(n,n)} \\
\text{>> } & \text{ op } = \text{lambda} \text{ X: A*X - X*S.T} \\
\text{>> } & \text{ s } = 4 \\
\text{>> } & \text{ X } = \text{idrs(op,B,s)}
\end{align*}
\]

http://ta.twi.tudelft.nl/nw/users/gijzen/IDR.html