MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

KONSENS

Federal Ministry
of Education
and Research

Complexity Reduction for Power Flow Simulations

M. Baumann S. Grundel P. Mlinarić

Motivation

The *energy transition* in Germany comes with new challenges for the mathematical modeling of power grids. Most prominently, this includes renewable energy resources that are modeled as time-dependent generators. Moreover, newly developed storage units can be added to the grid in order to improve network stability.

The simulation of the dynamical part of the power system is accelerated by means of Model Order Reduction (MOR),

$$\dot{x} = f(x(t)) \quad \rightsquigarrow \quad \dot{\hat{x}} = W_r^T f(V_r \hat{x}(t)), \tag{1}$$

where biorthogonal projection matrices V_r and W_r are used.

Optimal power flow

Power flow equations: An electrical network is modeled as a mathematical graph. The power flow at node i is described by the complex power $S_i = P_i + jQ_i$ and the complex voltage $V_i = |V_i|e^{j\delta_i}$ governed by the (stationary) *power flow equations*,

$$c_E(x(t)) = \begin{bmatrix} P_i - |V_i| \sum_{k=1}^N |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \\ Q_i - |V_i| \sum_{k=1}^N |V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \end{bmatrix} \overset{!}{=} 0,$$

for $x_i = (|V_i|, \delta_i, P_i, Q_i)$, $\delta_{ik} = \delta_i - \delta_k$, at all times t.

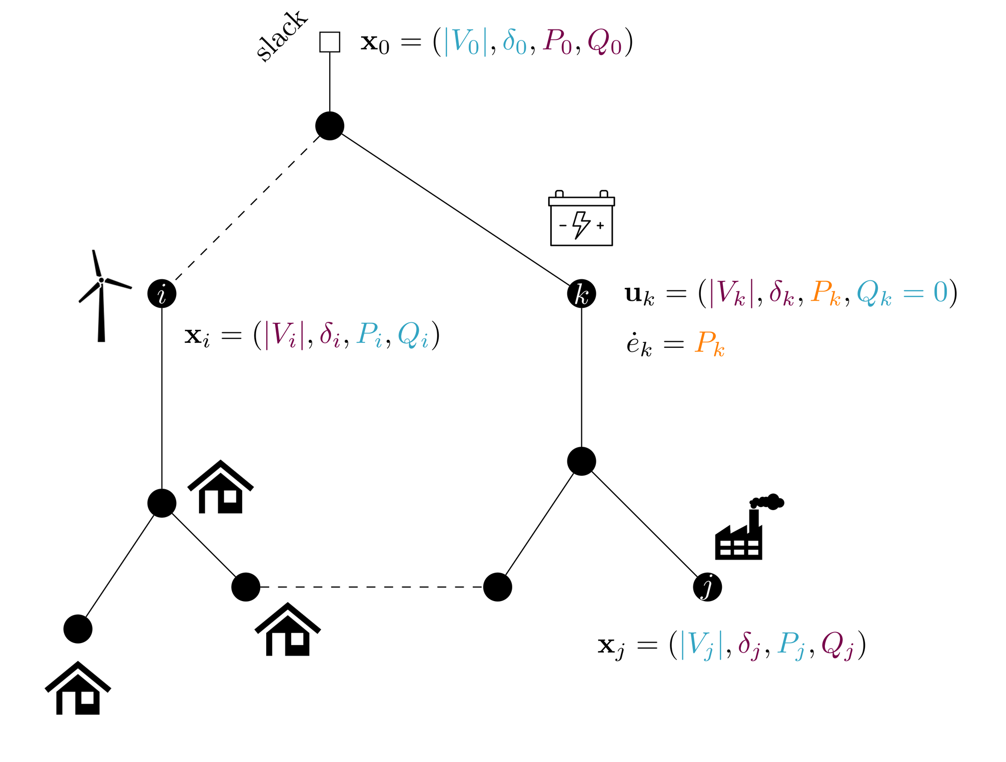

Figure 1: Overview of different components in a power grid. The colors indicate if a nodal variable is known, unknown, or a control.

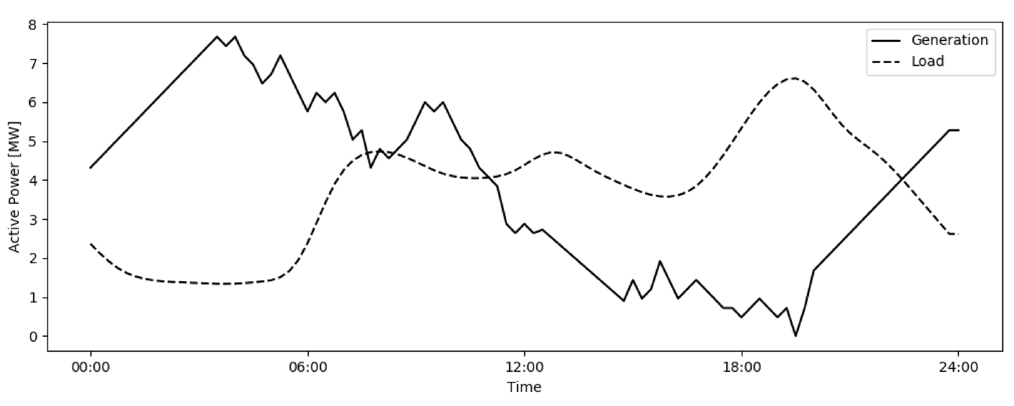

Figure 2: Mismatch of generation and load in a 24-hour time series.

Battery controls: We introduce storage units at some nodes of the power grid in order to improve the line loading in the network. The reactive power of these batteries are the control quantities u. The following optimal control problem is derived,

$$\begin{aligned}
&\underset{u}{\text{minimize}} && \int_{t_0}^{t_e} \ell(x(u), t)\, dt && \text{(line loading)} \\
&\text{subject to} && c_E(x(t)) + Bu(t) = 0 && \text{(network state)} \\
& && \dot{e}_k = u_k,\ e_k(0) = e_k^0 && \text{(battery state-of-charge)} \\
& && x_{\min} \le x_i(t) \le x_{\max} && \text{(network constraints)} \\
& && u_{\min} \le u_k(t) \le u_{\max} && \text{(control constraints)} \\
& && e_{\min} \le e_k(t) \le e_{\max} && \text{(battery constraints)}
\end{aligned}$$

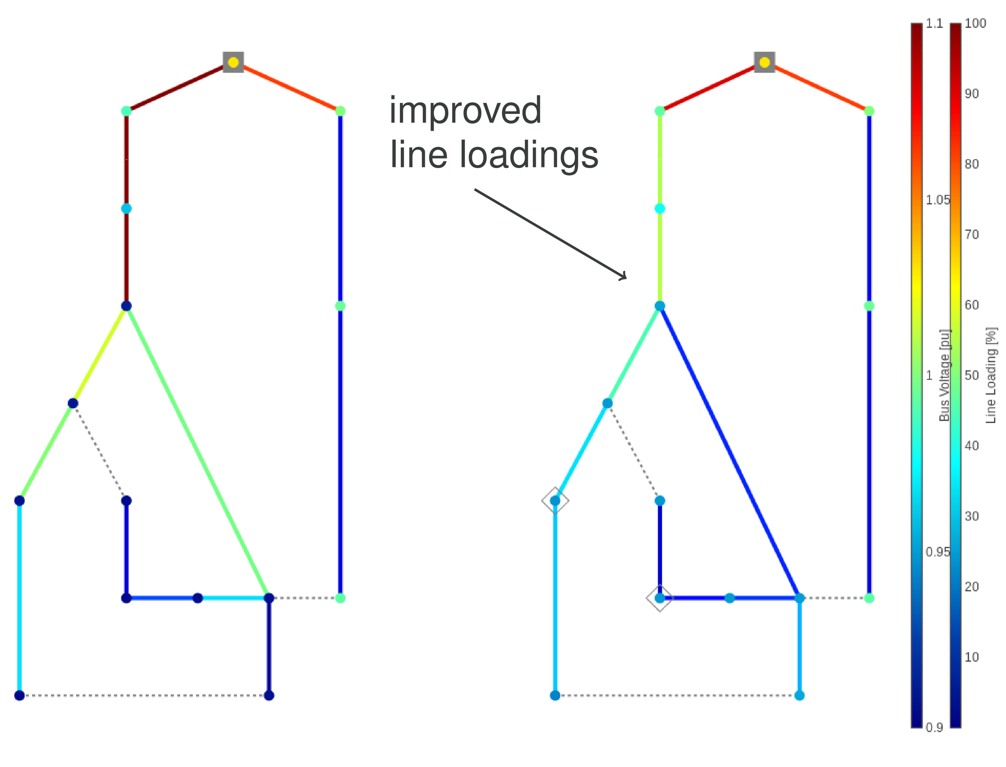

Figure 3: Improved line loadings in the CIGRE Task Force C6.04.02 network, with batteries located at '⋄'. Visualization using Python's `pandapower`.

Network clustering

Graph clustering [2] can be realized using Galerkin projection

$$V_r = W_r = P(\pi) \quad \text{in (1)},$$

where $P(\pi)$ is a characteristic matrix of a partition π, e.g., for $\pi = \{\{1,2,3,4\},\{5,6\},\{7\},\{8\},\{9,10\}\}$ we have

$$P(\pi) = \begin{bmatrix} 1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&1 \end{bmatrix}^T.$$

Then, if $\mathcal{A} = [a_{ij}]$ is the adjacency matrix of the original graph, the adjacency matrix of the reduced graph is $\widehat{\mathcal{A}} = P(\pi)^T \mathcal{A} P(\pi)$.

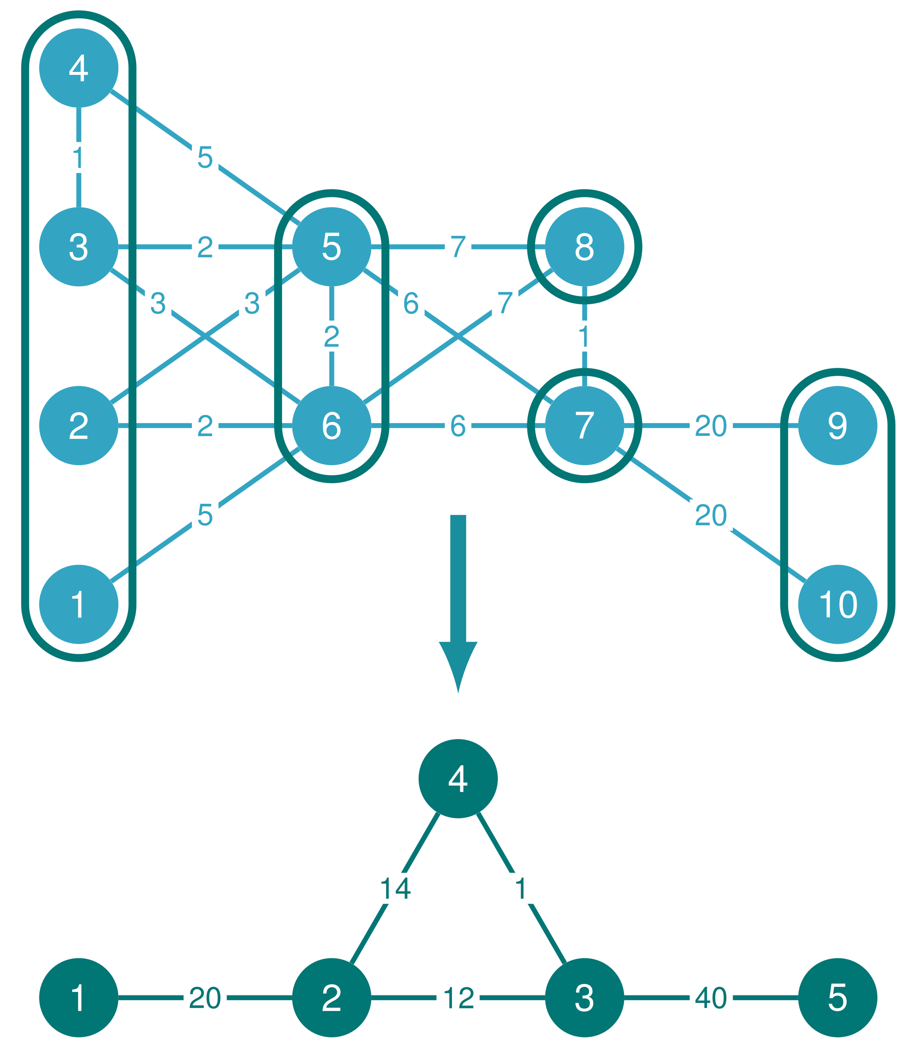

Figure 4: Reduction of a graph using clustering.

Dynamic power flow simulation

A network of generators is modeled using swing equations [3]

$$M_i \ddot{\delta}_i(t) + D_i \dot{\delta}_i(t) = P_i - \sum_{j=1}^{N} a_{ij} \sin(\delta_i(t) - \delta_j(t)), \tag{2}$$

where M_i, D_i, and P_i are respectively the inertia, damping, and power of the ith generator. The reduced-order model (ROM) we present is derived from a two-step clustering algorithm [2]:

1. Collect the leading modes from Proper Orthogonal Decomposition (POD) applied to the snapshots of δ_i and $\dot{\delta}_i$ in (2).
2. Apply the k-means clustering algorithm to the set of rows of POD's projection matrix.

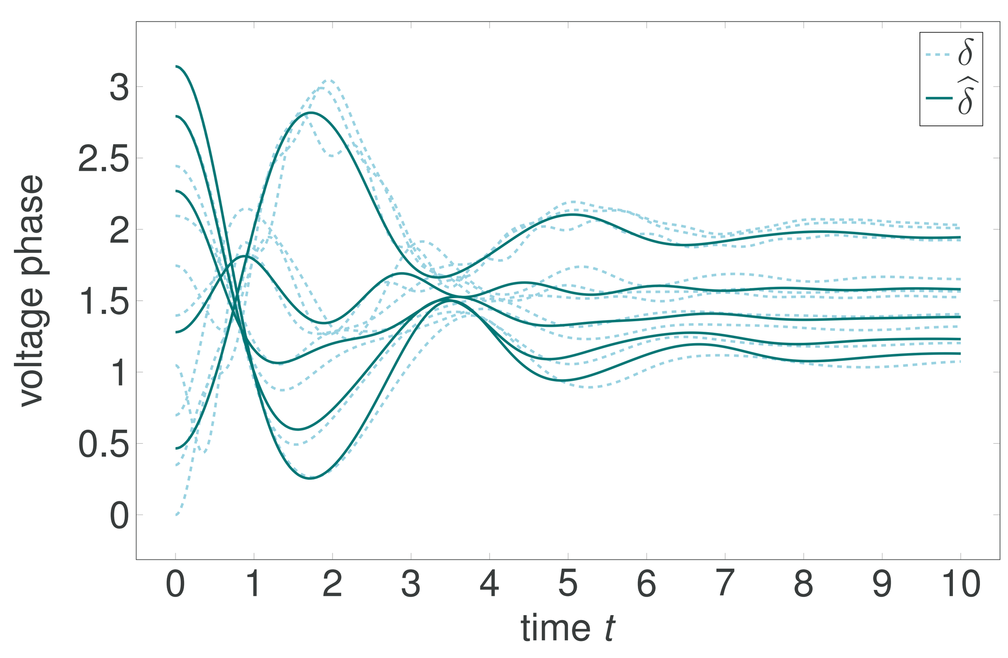

Figure 5: Simulation of the original swing equation (dashed lines) and its clustering-based ROM (solid lines).

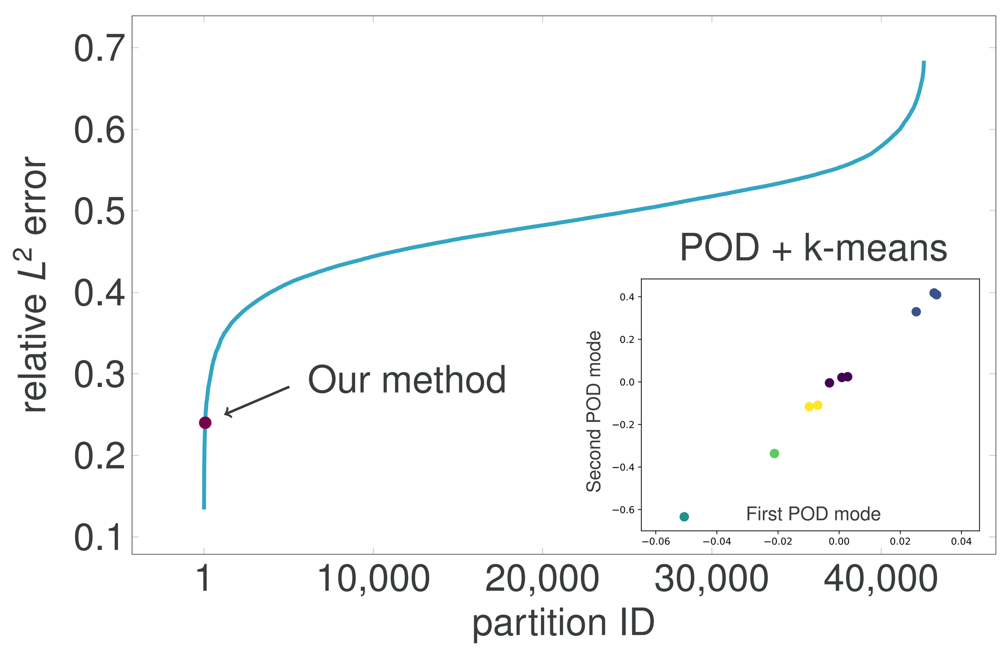

Figure 6: Relative error of all possible five-cluster partitions.

Power flow in quadratic form

The swing equations (2) can be written in quadratic form when introducing the elongated state variable $x_i = (\delta_i, \dot{\delta}_i, \sin\delta_i, \cos\delta_i)$,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + H\, x(t) \otimes x(t) + Bu(t), \\ y(t) &= Cx(t), \quad x(0) = x_0. \end{aligned} \tag{3}$$

In [1], the balanced truncation method is extended to derive matrices $(\widehat{A}, \widehat{B}, \widehat{C}, \widehat{H})$ of a reduced-order quadratic system, see Figure 7.

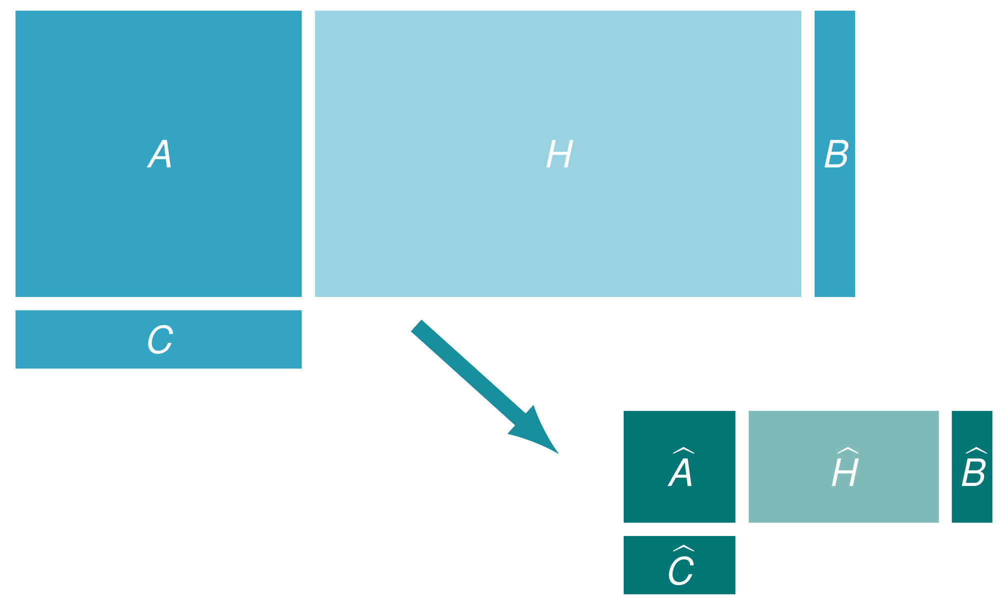

Figure 7: Illustration of the Petrov-Galerkin projections $\widehat{A} = W_r^T A V_r$, $\widehat{B} = W_r^T B$, $\widehat{C} = CV_r$, and $\widehat{H} = W_r^T H (V_r \otimes V_r)$.

As in the linear case, the projection matrices V_r and W_r are computed from the Gramians.

$$\begin{aligned} &\text{Solve} && A\mathbf{P} + \mathbf{P}A^T + H(\mathbf{P} \otimes \mathbf{P})H^T = -BB^T, \\ &\text{and} && A^T\mathbf{Q} + \mathbf{Q}A + H^{(2)}(\mathbf{P} \otimes \mathbf{Q})(H^{(2)})^T = -C^T C, \end{aligned}$$

for the controllability and observability Gramians $\mathbf{P}$ and $\mathbf{Q}$.

Future work

Fault recovery: When transmission line failure occurs at time t_s, the topology of the network changes, which means that the dynamics is described by a different set of matrices in (3):

$$(A_1, B_1, C_1, H_1) \overset{t_s}{\rightsquigarrow} (A_2, B_2, C_2, H_2).$$

In [4], linear switched systems are considered, i.e., the case when $H_1 = H_2 = 0$ for all t.

Reduced-order models for power flow simulation:

- Derive ROMs for distribution grid that are of sufficient accuracy on higher grid levels.
- Error bounds and quantification of uncertainties.
- Exploit grid flexibilities.

References

[1] P. Benner and P. Goyal, *Balanced truncation model order reduction for quadratic-bilinear systems*, e-prints 1705.00160, arXiv, 2017.
[2] P. Mlinarić, S. Grundel, and P. Benner, *Efficient model order reduction for multi-agent systems using QR decomposition-based clustering*, in 54th IEEE Conference on Decision and Control (CDC), Osaka, Japan, Dec. 2015, pp. 4794–4799.
[3] P. Mlinarić, T. Ishizaki, A. Chakrabortty, S. Grundel, P. Benner, and J.-I. Imura, *Synchronization and aggregation of nonlinear power systems with consideration of bus network structures*, in Proc. European Control Conf. (ECC), 2018.
[4] I. Pontes Duff, S. Grundel, and P. Benner, *New Gramians for linear switched systems: Reachability, observability, and model reduction*, e-print 1806.00406, arXiv, 2018.

The KONSENS project

As part of the initiative *Mathematics for Innovation* by the Federal Ministry of Education and Research, the project "Konsistente Optimierung und Stabilisierung elektrischer Netzwerksysteme" (KONSENS) addresses mathematical challenges arising from the energy transition in Germany.

CSC Members	M. Baumann, S. Grundel
TU Chemnitz	C. Helmberg et al., S. Streif et al.
TU Ilmenau	K. Worthmann et al.
Industry Partner	Energy Saxony e.V., TenneT TSO GmbH, ENSO NETZ GmbH, Venios GmbH