Nonlinear Model Order Reduction using POD/DEIM for Optimal Control of Burgers' Equation

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Outline

1. What is Model Order Reduction (MOR) ?
2. Model Order Reduction using POD-DEIM
   - Proper Orthogonal Decomposition (POD)
   - Discrete Empirical Interpolation Method (DEIM)
   - Application: MOR for Burgers' equation
3. PDE-constrained Optimization
   - Second-order optimization algorithm
   - First-order methods: BFGS and SPG
4. Optimal Control for the reduced-order Burgers' equation
5. Summary and future research

Nonlinear dynamical systems

Consider the nonlinear dynamical system

$$\begin{aligned} \dot{\mathbf{y}}(t) &= A\mathbf{y}(t) + \mathbf{F}(t, \mathbf{y}(t)), \quad \mathbf{y}(t) \in \mathbb{R}^N \\ \mathbf{y}(0) &= \mathbf{y}_0 \end{aligned} \tag{1}$$

- arises in many applications, e.g. mechanical systems, fluid dynamics, neuron modeling, ...
- the matrix A represents the linear dynamical behavior and the function $\mathbf{F}$ represents nonlinear dynamics
- often large dimension of (1) leads to *huge* computational work

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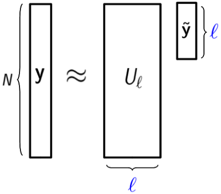
The idea of model order reduction

Approximate the state via

$$\mathbf{y}(t) \approx U_\ell \tilde{\mathbf{y}}(t), \quad U_\ell \in \mathbb{R}^{N \times \ell}, \tilde{\mathbf{y}} \in \mathbb{R}^{\ell},$$

where the matrix U_ℓ has orthonormal columns, the so-called *principal components* of $\mathbf{y}$, and $\ell \ll N$.



Galerkin projection of the original full-order system leads to a reduced system of ℓ equations:

$$U_\ell^T \left[U_\ell \dot{\tilde{\mathbf{y}}} - A U_\ell \tilde{\mathbf{y}} - \mathbf{F}(t, U_\ell \tilde{\mathbf{y}}) \right] = 0$$

$$\Rightarrow \quad \dot{\tilde{\mathbf{y}}} = \underbrace{U_\ell^T A U_\ell}_{=:\tilde{A}} \tilde{\mathbf{y}} + U_\ell^T \mathbf{F}(t, U_\ell \tilde{\mathbf{y}})$$

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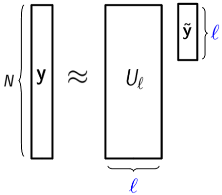

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Two questions are left...

Considering the reduced model

$$\dot{\tilde{\mathbf{y}}}(t) = \tilde{A}\tilde{\mathbf{y}}(t) + U_\ell^T \mathbf{F}(t, U_\ell \tilde{\mathbf{y}}(t)), \quad \tilde{\mathbf{y}}(t) \in \mathbb{R}^\ell$$

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1. How to obtain the matrix U_ℓ of principal components ?
2. Note that $U_\ell \tilde{\mathbf{y}}(t) \in \mathbb{R}^N$ is still large. How do we evaluate $\mathbf{F}(t, U_\ell \tilde{\mathbf{y}}(t))$ efficiently ?

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The Proper Orthogonal Decomposition (POD)

During the numerical simulation, build up the snapshot matrix

$$Y := [\mathbf{y}(t_1), \dots, \mathbf{y}(t_{n_s})] \in \mathbb{R}^{N \times n_s},$$

with n_s being the number of snapshots.

Perform a Singular Value Decomposition (SVD)

$$Y = U \Sigma V^T$$

and let $U_\ell := \texttt{U(:,1:l)}$ consist of those left singular vectors of Y that correspond to the ℓ largest singular values in Σ.

The Discrete Empirical Interpolation Method (DEIM)

Consider the nonlinearity

$$\mathbf{N} := \underbrace{U_\ell^T}_{\ell\times N} \underbrace{\mathbf{F}(t, U_\ell \tilde{\mathbf{y}}(t))}_{N\times 1}$$

The approximation

$$\mathbf{F} \approx W\mathbf{c}, \quad W \in \mathbb{R}^{N\times m}, \mathbf{c} \in \mathbb{R}^m$$

is over-determined. Therefore, find projector $\mathcal{P}$ such that:

$$\begin{aligned} \mathcal{P}^T\mathbf{F} = (\mathcal{P}^T W)\mathbf{c} \quad &\Rightarrow \quad \mathbf{F} \approx W\mathbf{c} = W(\mathcal{P}^T W)^{-1}\mathcal{P}^T\mathbf{F} \\ &\Rightarrow \quad \mathbf{N} \approx U_\ell^T W \underbrace{(\mathcal{P}^T W)}_{m\times m}{}^{-1}\mathcal{P}^T\mathbf{F}(t, U_\ell\tilde{\mathbf{y}}(t)) \end{aligned}$$

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Algorithm 1 The DEIM algorithm [Chaturantabut, Sorensen, 2010]

INPUT: $\{\mathbf{w}_i\}_{i=1}^m \subset \mathbb{R}^N$ linear independent
OUTPUT: $\vec{\wp} = [\wp_1, \dots, \wp_m]^T \in \mathbb{R}^m, \ \mathcal{P} \in \mathbb{R}^{N \times m}$
$[|\rho|, \wp_1] = \max\{|\mathbf{w}_1|\}$
$W = [\mathbf{w}_1], \mathcal{P} = [\mathbf{e}_{\wp_1}], \vec{\wp} = [\wp_1]$
for $i = 2$ to m **do**
  Solve $(\mathcal{P}^T W)\mathbf{c} = \mathcal{P}^T \mathbf{w}_i$ for $\mathbf{c}$
  $\mathbf{r} = \mathbf{w}_i - W\mathbf{c}$
  $[|\rho|, \wp_i] = \max\{|\mathbf{r}|\}$
  $W \leftarrow [W \ \mathbf{w}_i], \mathcal{P} \leftarrow [\mathcal{P} \ \mathbf{e}_{\wp_i}], \vec{\wp} \leftarrow \begin{bmatrix} \vec{\wp} \\ \wp_i \end{bmatrix}$
end for

The product $\mathcal{P}^T\mathbf{F}$ is a selection of entries

Let $m = 3$. Suppose the DEIM-algorithm has chosen indices $\wp_1, \dots, \wp_m$ such that:

$$\mathcal{P}^T\mathbf{F} = \begin{array}{c} \\ \\ \begin{bmatrix} 0 & \dots & 1 & \dots & 0 \\ 0 & 1 & \dots & \dots & 0 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \\ \\ \end{array} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix} = \begin{bmatrix} F_{\wp_1} \\ F_{\wp_2} \\ F_{\wp_m} \end{bmatrix}$$

$$\begin{array}{ccccc} & & \wp 1 & & \\ & & \downarrow & & \\ 0 & \dots & 1 & \dots & 0 \\ 0 & 1 & \dots & \dots & 0 \\ 0 & \dots & 0 & 1 & 0 \\ & & & \uparrow & \\ & & & \wp m & \end{array}$$

Assuming that $\mathbf{F}(\cdot)$ acts pointwise, we obtain:

$$\begin{aligned} \mathbf{N} &\approx U_\ell^T W(\mathcal{P}^T W)^{-1}\mathcal{P}^T\mathbf{F}(t, U_\ell\tilde{\mathbf{y}}(t)) \\ &= \underbrace{U_\ell^T W(\mathcal{P}^T W)^{-1}}_{\ell\times m}\underbrace{\mathbf{F}(t, \mathcal{P}^T U_\ell\tilde{\mathbf{y}}(t))}_{m\times 1} \end{aligned}$$

The nonlinear 1D Burgers' model

$$
\begin{aligned}
y_t + \left(\frac{1}{2}y^2 - \nu y_x\right)_x &= f, \quad (x,t) \in (0,L)\times(0,T),\\
y(t,0) = y(t,L) &= 0, \quad t \in (0,T),\\
y(0,x) &= y_0(x), \quad x \in (0,L).
\end{aligned}
$$

1. FEM-discretization in space leads to:

$$
\begin{aligned}
M\dot{\mathbf{y}}(t) &= -\frac{1}{2}B\mathbf{y}^2(t) - \nu C\mathbf{y}(t) + \mathbf{f}, \quad t > 0\\
\mathbf{y}(0) &= \mathbf{y}_0
\end{aligned}
$$

2. Time integration via implicit Euler + Newton's method

POD-DEIM for Burgers' equation

Suppose, Φ_ℓ is an M-orthogonal POD basis.

The POD reduced Burgers' equation

$$\overbrace{\Phi_\ell^T M \Phi_\ell}^{=I_\ell} \dot{\tilde{y}}(t) = -\frac{1}{2}\Phi_\ell^T B(\Phi_\ell \tilde{\mathbf{y}}(t))^2 - \nu \Phi_\ell^T C \Phi_\ell \tilde{\mathbf{y}}(t)$$

$$\Rightarrow \quad \dot{\tilde{y}}(t) = -\frac{1}{2} B_\ell (\Phi_\ell \tilde{\mathbf{y}}(t))^2 - \nu C_\ell \tilde{\mathbf{y}}(t)$$

Next, obtain W via a truncated SVD of $[\mathbf{y}^2(t_1), \ldots, \mathbf{y}^2(t_{n_s})]$ and apply DEIM to the columns of W.

The POD-DEIM reduced Burgers' equation

$$\dot{\tilde{y}}(t) = -\frac{1}{2}\tilde{B}(\tilde{F}\tilde{\mathbf{y}}(t))^2 - \nu \tilde{C}\tilde{\mathbf{y}}(t),$$

with $\tilde{B} = \Phi_\ell^T B W (\mathcal{P}^T W)^{-1} \in \mathbb{R}^{\ell \times m}$, $\tilde{F} = \mathcal{P}^T \Phi_\ell \in \mathbb{R}^{m \times \ell}$, and $\tilde{C} = C_\ell \in \mathbb{R}^{\ell \times \ell}$.

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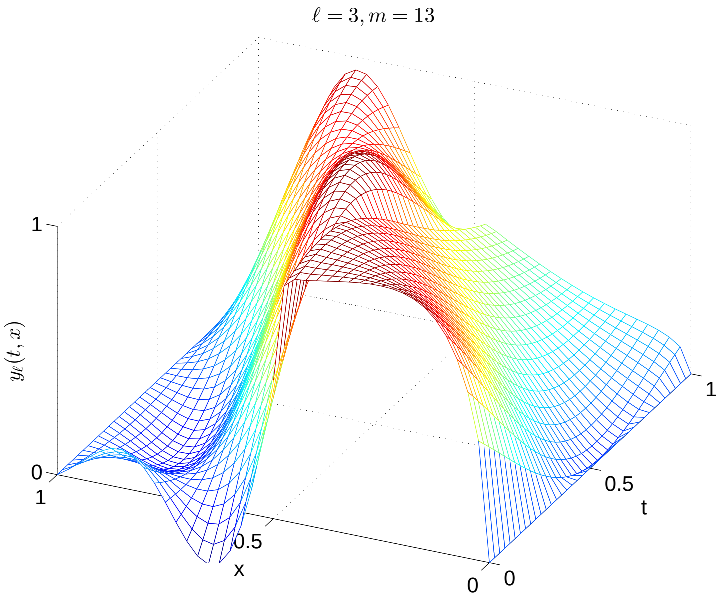
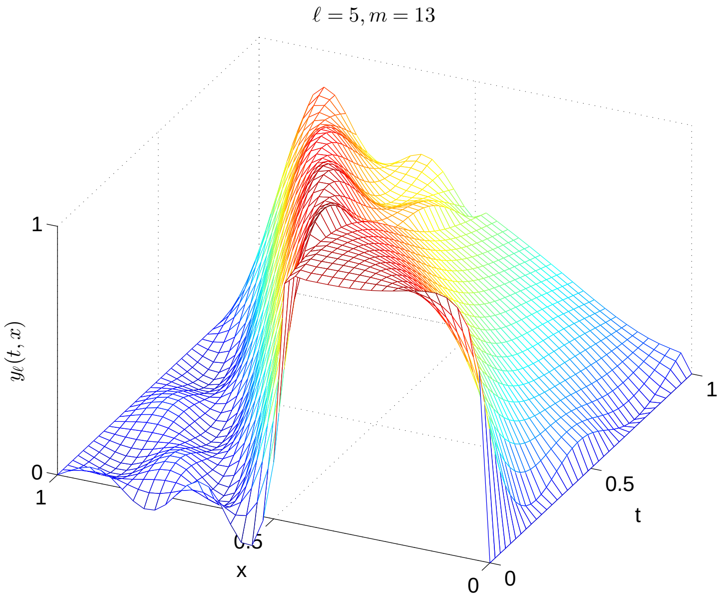

$\ell = 3, m = 13$
$y_\ell(t,x)$
1
0
1
0.5
x
0
0
0.5
t
1

Application: MOR for Burgers' equation



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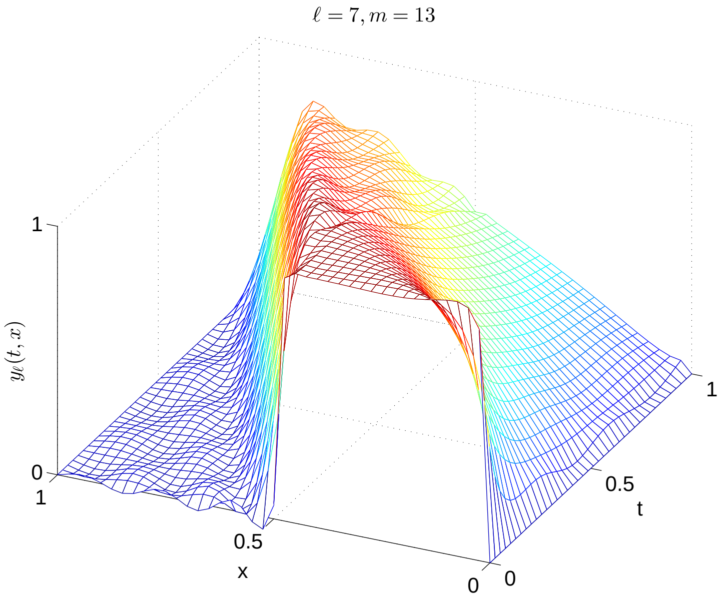

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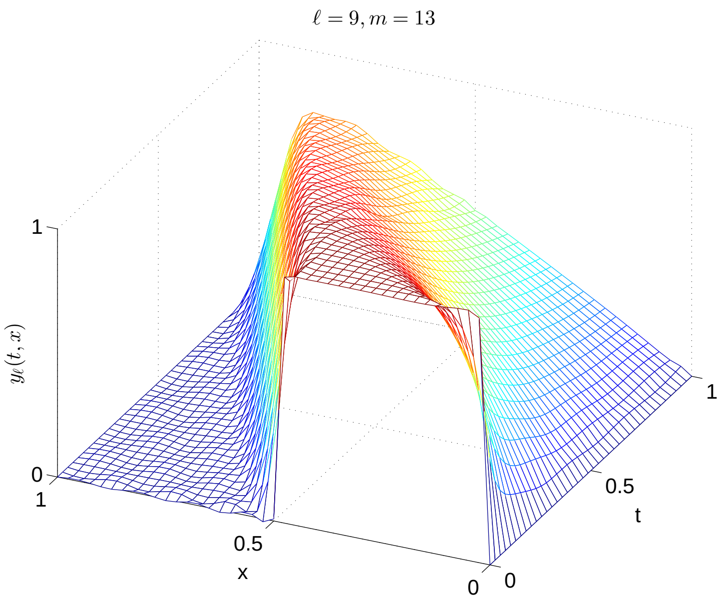

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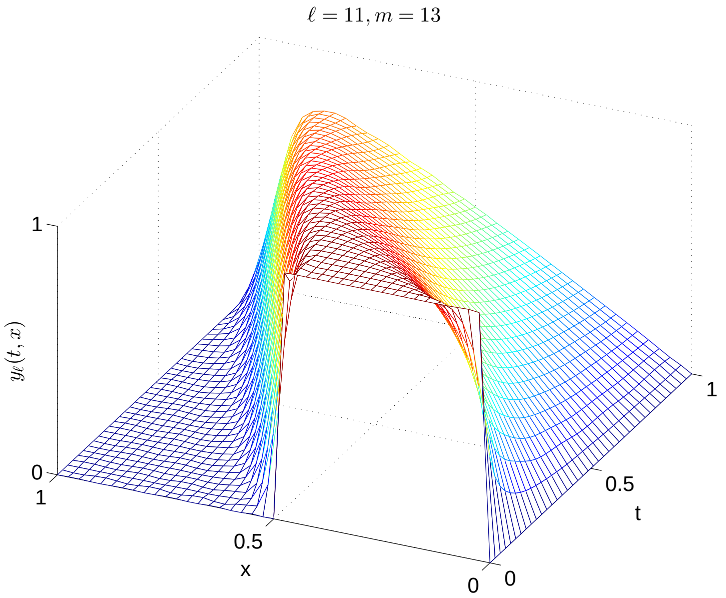
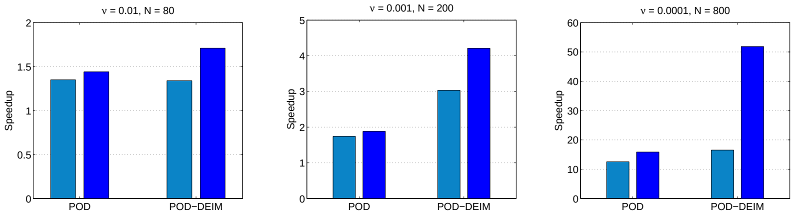


Computational Speedup [1]

Conclusion: High accuracy of the POD-DEIM reduced model.

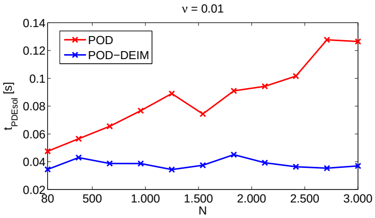
But is it also faster?

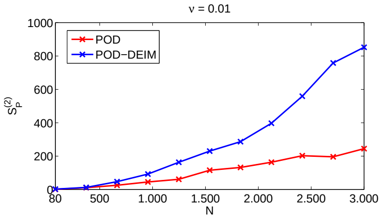


- Spatial discretization of the full model depends on viscosity parameter ν
- choose ℓ, m such that relative L_2-error in $\mathcal{O}(10^{-4})$

Computational Speedup [2]

For a fixed $\nu = 0.01$, we could show the independence of the POD-DEIM reduced model of the full-order dimension N.





- Computation time for solving the POD-DEIM reduced Burgers' equation is almost constant (right)
- POD-DEIM almost 4 times faster than pure POD (left)

PDE-constrained optimization

Minimize

$$\min_u \mathcal{J}(y(u), u),$$

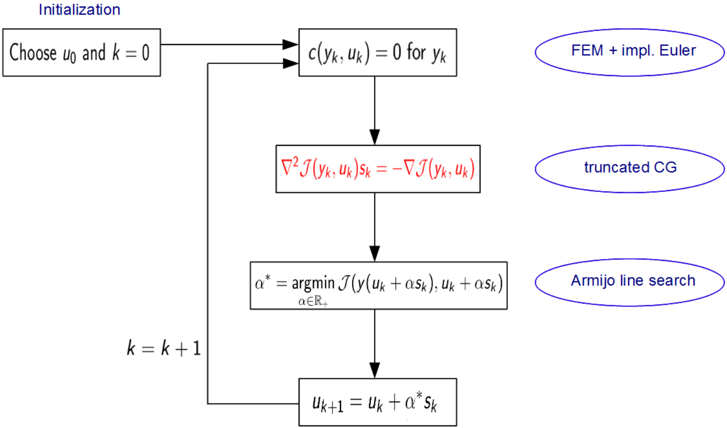
where y is the solution to a nonlinear, possibly time-dependent partial differential equation,

$$c(y, u) = 0.$$

- $\mathcal{J}$ is called objective function,
- in order to evaluate $\mathcal{J}$, we need to solve $c(y, u) = 0$ for $y(u)$,
- solve with algorithms for unconstrained minimization problems.

A Newton-type optimization algorithm

Minimize $\mathcal{J}(y(u), u)$ in u using information of the first and second derivative.



Gradient computation via adjoints

Consider the Lagrangian function

$$\mathcal{L}(y,u,\lambda) = \mathcal{J}(y,u) + \lambda^T c(y,u)$$

and **impose** the zero-gradient condition $\nabla_y \mathcal{L}(y,u,\lambda) = 0$.

We derive the *adjoint equation*:

$$c_y(y(u),u)^T \lambda = -\nabla_y \mathcal{J}(y(u),u)$$

Algorithm 2 Computing $\nabla \hat{\mathcal{J}}(u)$ via adjoints [Heinkenschloss, 2008]

1: For a given control u, solve $c(y,u) = 0$ for the state $y(u)$
2: Solve the adjoint equation $c_y(y(u),u)^T \lambda = -\nabla_y \mathcal{J}(y(u),u)$ for $\lambda(u)$
3: Compute $\nabla \hat{\mathcal{J}}(u) = \nabla_u \mathcal{J}(y(u),u) + c_u(y(u),u)^T \lambda(u)$

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First-order optimization algorithms

Instead of solving

$$\nabla^2 \mathcal{J}(y_k, u_k) s_k = -\nabla \mathcal{J}(y_k, u_k),$$

first-order methods approximate the Hessian via H_k and solve

$$H_k s_k = -\nabla \mathcal{J}(y_k, u_k).$$

- We have used Matlab implementations of the BFGS and the SPG method,
- Evaluation of $\mathcal{J}$ and gradient computation as seen before,
- SPG easily allows to include bounds on the control, i.e. $u_{lower} \leq u(t,x) \leq u_{upper}$ which is used in many applications

Optimal Control problem for Burgers' equation

Minimize

$$\min_{u} \frac{1}{2} \int_0^T \int_0^L [y(t,x) - z(t,x)]^2 + \omega u^2(t,x)\; dx\; dt,$$

where y is a solution to the nonlinear Burgers' equation

$$\begin{aligned} y_t + \left(\frac{1}{2}y^2 - \nu y_x\right)_x &= f + u, \quad && (x,t) \in (0,L) \times (0,T), \\ y(t,0) = y(t,L) &= 0, && t \in (0,T), \\ y(0,x) &= y_0(x), && x \in (0,L). \end{aligned}$$

- u is the control that determines y
- z is the desired state

Control goal

We want to control the solution of Burgers' equation in such a way that it stays in the desired state $z(t,\cdot) = y_0, \forall t$:

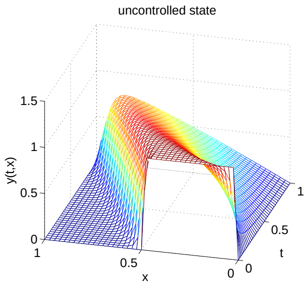

Figure: Uncontrolled ($u \equiv 0$) and desired state for $\nu = 0.01$.

Numerical treatment

1. Discretize the objective function and Burgers' equation in time and space
2. Apply adjoints in order to compute gradient and Hessian
3. Apply first-order or second-order optimization algorithm
4. Explore the usage of a POD-DEIM reduced model

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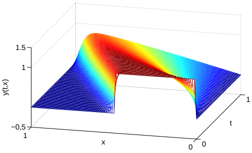
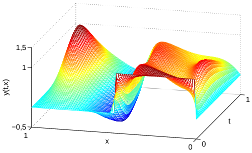
Numerical tests

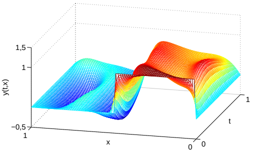
Newton-type method for the full-order Burgers' model:



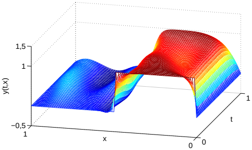
$k = 0$ (uncontrolled)



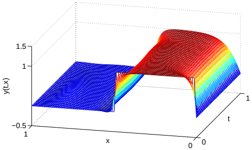
$k = 1$



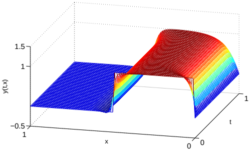
$k = 2$



$k = 3$

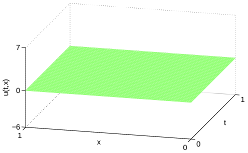


$k = 4$

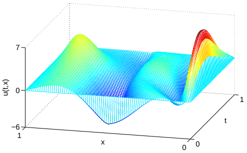


$k = 5$

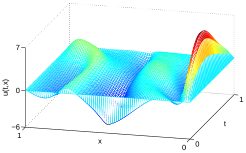
The corresponding optimal control at each iteration:



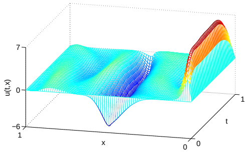
$k = 0$ (initial)



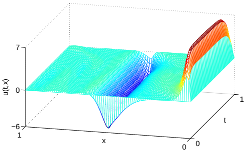
$k = 1$



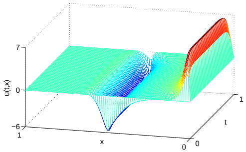
$k = 2$



$k = 3$

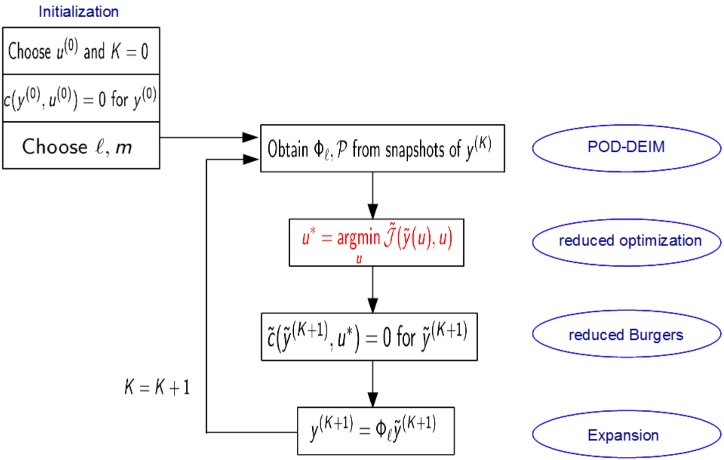


$k = 4$

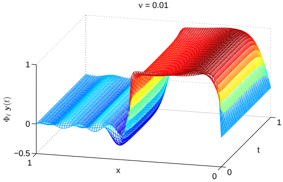


$k = 5$

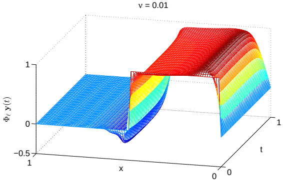
We propose the following algorithm for POD-DEIM reduced optimal control :



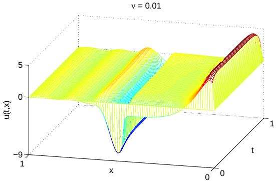
Final state and control of the POD-DEIM reduced optimal control problem:



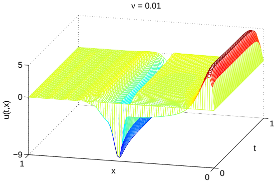
$\ell = m = 7$



$\ell = m = 15$



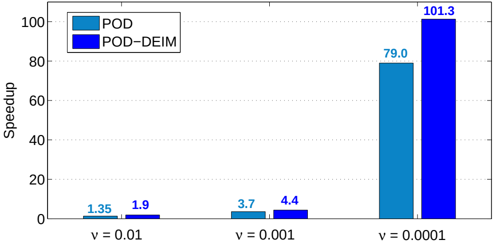
$\ell = m = 7$



$\ell = m = 15$

Computational Speedup [3]

Reduced optimal control using the Newton-type method:

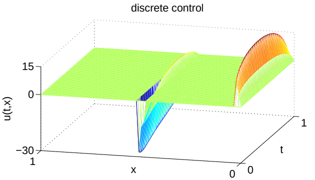


- at final state: relative L_2-error in $\mathcal{O}(10^{-2})$
- comparable value of the objective function at convergence
- use same stopping criteria for full-order and reduced model

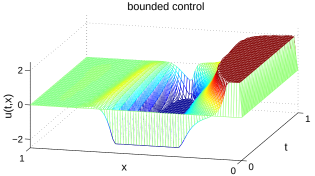
Computational Speedup [4]

Some other results.

For $\nu = 0.0001$, low-dimensional control leads to a speedup of ~ 20 for all three optimization methods.



SPG allows a bounded control $-2 \leq u(t,x) \leq 2$. For $\nu = 0.0001$, we obtained a speedup of 3.6 for POD and 8.8 for POD-DEIM.



Concluding Remarks

What I learnt:

- The accuracy of the reduced Burgers' model is of the same order when POD is extended by DEIM.
- Optimal Control of Burgers' equation using POD-DEIM leads to a speedup of ~ 100 for small ν.
- For the reduced model, all derivatives need to be computed in terms of the reduced variable. This can be quite hard in practice.

Future Research

What I still want to learn:

- Use the POD basis Φ_ℓ also for dimension reduction of the control, i.e.

$$\mathbf{u}(t) \approx \Phi_\ell \tilde{\mathbf{u}}(t) = \sum_{i=1}^{\ell} \varphi_i \tilde{u}_i(t)$$

- Extend Burgers' model to 2D/3D
- More sophisticated choice of reduced dimensions ℓ and m

This Master project was supervised by
Marielba Rojas and Martin van Gijzen.

Thank you for your attention!

Are there any questions or remarks?

https://github.com/ManuelMBaumann/MasterThesis

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Further information can be found in...

- S. Chaturantabut and D. Sorensen
Nonlinear Model Reduction via Discrete Empirical Interpolation.
SIAM Journal of Scientific Computing, 2010.

- M. Heinkenschloss
Numerical solution of implicitly constrained optimization problems.
Technical report, Department of Computational and Applied Mathematics, Rice University, 2008.

- K. Kunisch and S. Volkwein
Control of the Burgers Equation by a Reduced-Order Approach Using Proper Orthogonal Decomposition.
Journal of Optimization Theory and Applications, 1999.