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Nested Krylov methods for shifted linear systems

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Several applications require the solution of a sequence of shifted linear systems of the form

$$(A - \omega_k I)\mathbf{x}_k = \mathbf{b}, \tag{1}$$

where $A \in \mathbb{C}^{N\times N}, \mathbf{b} \in \mathbb{C}^N$, and $\{\omega_k\}_{k=1}^n \in \mathbb{C}$ is a sequence of shifts. For example, shifted linear systems arise in model order reduction as well as in the geophysical exploration of both acoustic and elastic waves.

In our application, we focus on wave propagation through elastic media in a frequency-domain formulation. This formulation has specific advantages when modeling visco-elastic effects. In order to improve the imaging of the earth crust, so-called *full waveform inversion* is computed which is an optimization problem at multiple wave frequencies. Therefore, the grid size must be small enough to describe the wave, which for high frequencies results in very large shifted linear systems of the form (1).

In principle, a sequence of shifted systems (1) can be solved almost at the cost of a single solve using so-called shifted Krylov methods. These methods exploit the property that Krylov subspaces are invariant under arbitrary diagonal shifts ω to the matrix A, i.e.,

$$\mathcal{K}_m(A, \mathbf{b}) \equiv \mathcal{K}_m(A - \omega I, \mathbf{b}), \quad \forall m \in \mathbb{N}, \forall \omega \in \mathbb{C}. \tag{2}$$

However, in practical applications, the preconditioning of (1) is required which in general destroys the shift-invariance property (2). In [1], a polynomial preconditioner that preserves the shift-invariance is suggested. The presented work is a new approach to the iterative solution of (1). We use *nested* Krylov methods that use an inner Krylov method as a preconditioner for an outer Krylov iteration. In order to deal with the shift-invariance, our algorithm only requires the inner Krylov method to produce collinear residuals for the shifted systems. In my talk, I will concentrate on two possible combinations of Krylov methods for the nested framework, namely FOM-GMRES and IDR-QMRIDR. Since residuals in multi-shift IDR are not collinear by default, the development of a collinear IDR variant which is suitable as an inner method in the new framework is a second main contribution of this work.

This is joint work with Martin B. van Gijzen [2].

References

[1] Mian Ilyas Ahmad, Daniel B. Szyld, and Martin B. van Gijzen. *Preconditioned multi-shift BiCG for $\mathcal{H}_2$-optimal model reduction.* Technical Report 12-06-15, Department of Mathematics, Temple University, 2012. Revised March 2013.

[2] Manuel Baumann and Martin B. van Gijzen. *Nested Krylov methods for shifted linear systems.* Technical Report 14-01, Delft Institute of Applied Mathematics, Delft University of Technology, 2014.

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